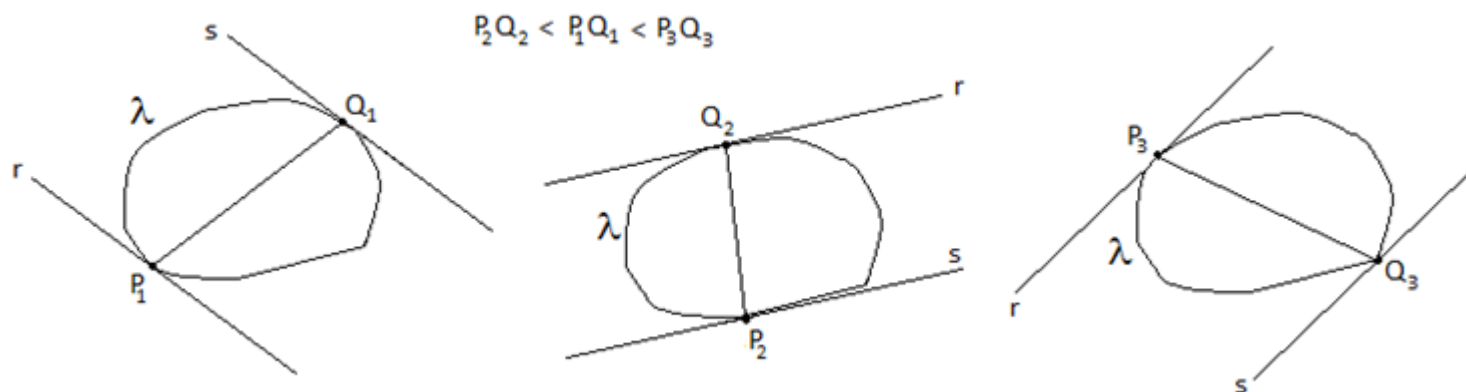


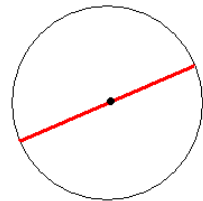
# Uma de definição de largura

Largura de uma curva fechada convexa é o valor máximo das distâncias entre dois pontos quaisquer da borda dessa curva.

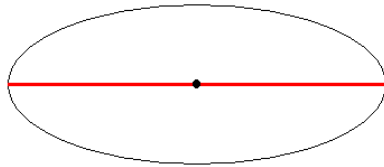


ver definição precisa em [1]

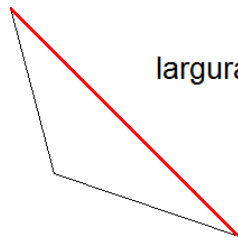
# Largura de algumas curvas



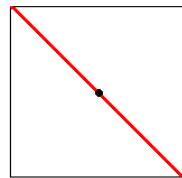
largura da circunferência → diâmetro



largura da elipse → eixo maior



largura de um triângulo qualquer → maior lado do triângulo



largura do quadrado → diagonal

# A função Pi

$Pi(\lambda) = \frac{\text{perímetro de } \lambda}{\text{largura de } \lambda}$  ,  $\lambda$  : curva fechada convexa

$$Pi(\text{circunferência}) = \pi \cong 3,14$$

$$Pi(\text{triângulo equilátero}) = 3$$

$$Pi(\text{triângulo retângulo isósceles}) = 1 + \sqrt{2} \cong 2,41$$

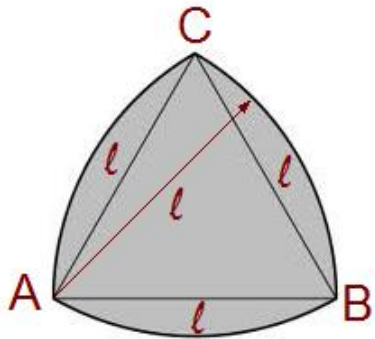
$$Pi(\text{quadrado}) = 2\sqrt{2} \cong 2,83$$

# Algumas boas perguntas

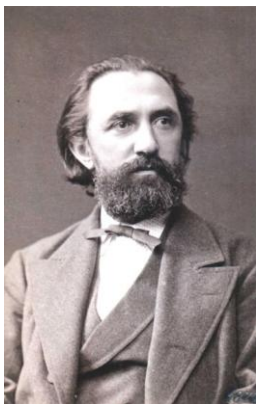
1. Qual é o maior valor possível de  $P_i$  (triângulo)?  
E o menor?
2. Qual é o maior valor possível de  $P_i$  (quadrilátero)?
3. A função  $P_i(\lambda)$  é injetora?
4. Existe  $\lambda$  tal que  $P_i(\lambda) > \pi$ ?

ver extensões e aprofundamentos dessas ideias em [2] e [3]

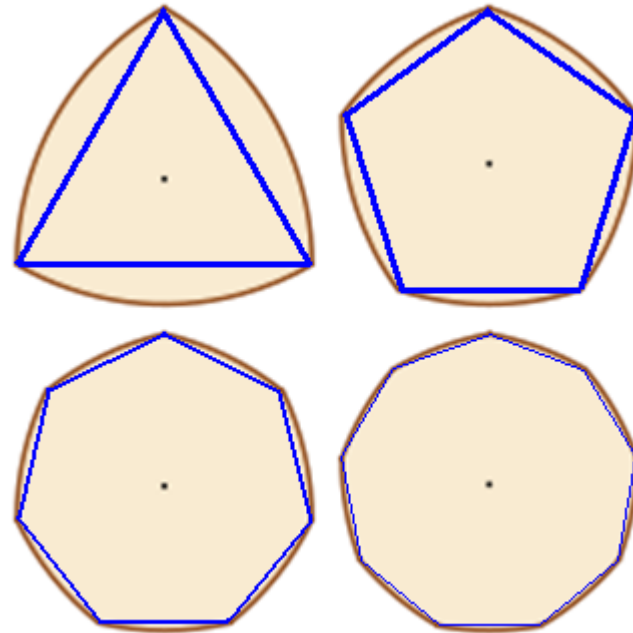
# Curvas de largura constante



Triângulo de Reuleaux

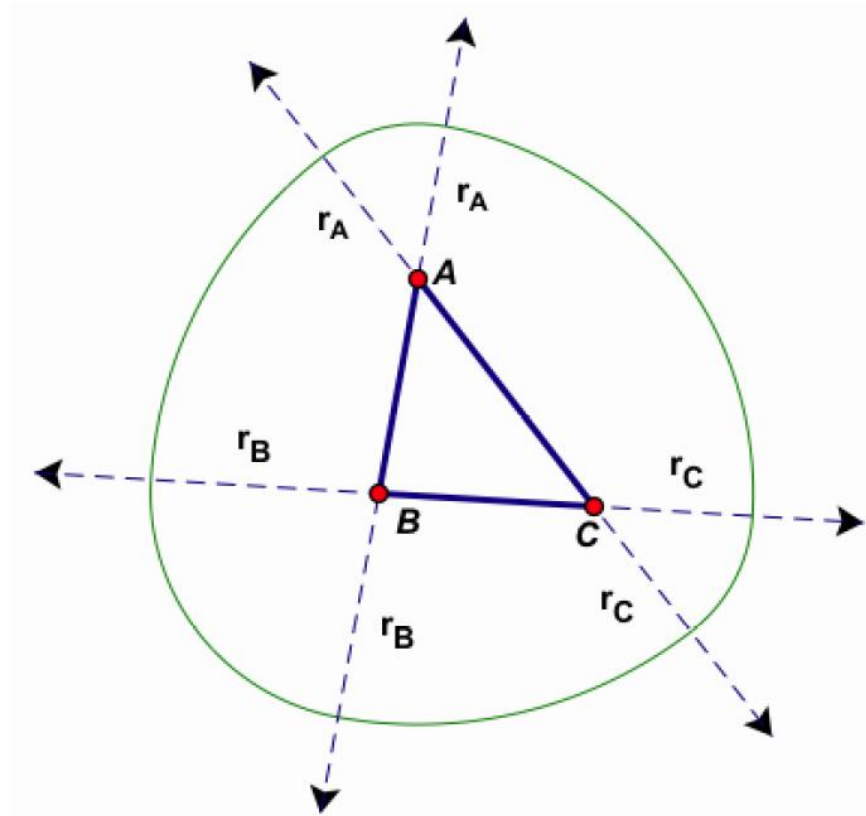


Franz Reuleaux  
(1829-1905)



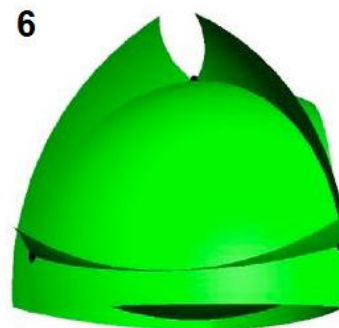
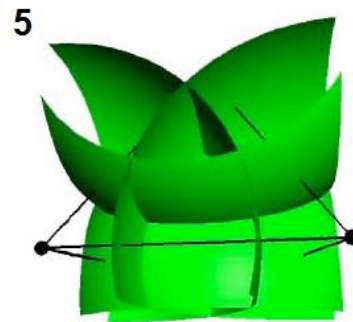
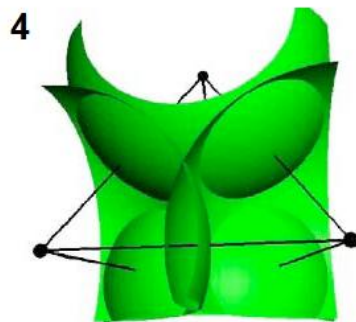
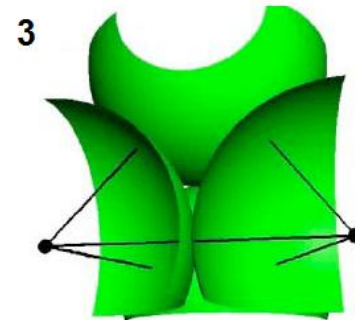
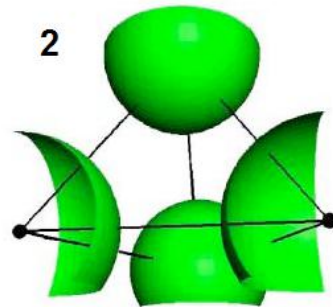
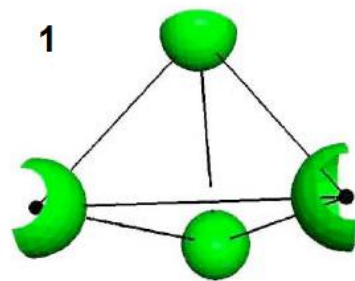
Polígonos de Reuleaux

# Reuleaux a partir de triângulo irregular

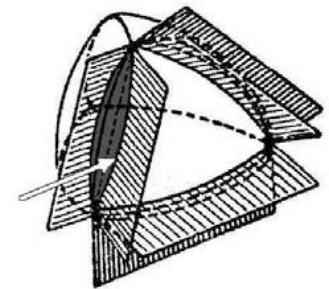


ver demonstração em [4]

# Surpresas na terceira dimensão...



**Tetraedro  
de Reuleaux**



# Largura constante na terceira dimensão

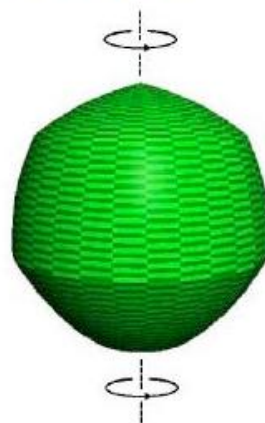
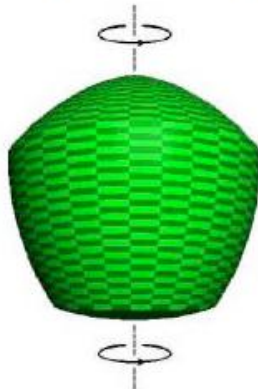
Sólido de Meissner



Ernst Meissner  
(1883-1939)

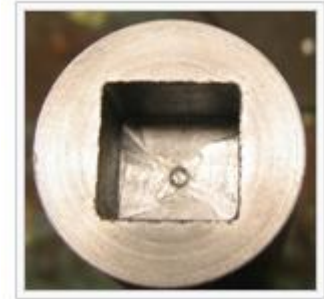
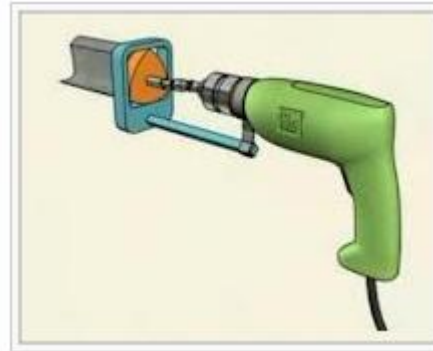
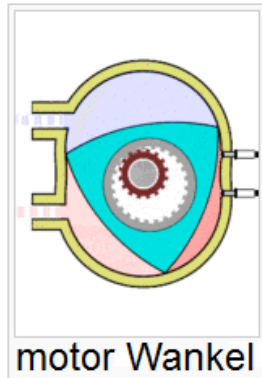
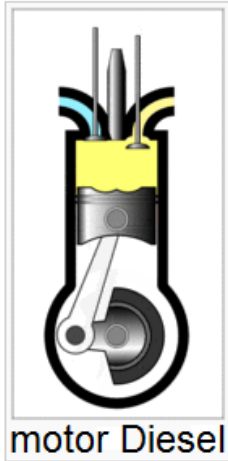
ver referência [6]

Sólidos de Reuleaux de revolução





# Aplicações das formas de Reuleaux



## Bibliografia (disponível na internet, exceto [5] e [7])

- [1] VOLOCH, J. F. ***Curvas de largura constante***. Matemática Universitária, no. 5, jun-1987, IMPA, RJ.
- [2] BALL, Derek G. ***A generalisation of  $\pi$*** . The Mathematical Gazette, vol. 57, no. 402, dec-1973.
- [3] GRIFFITHS, D., CULPIN, D. ***Pi-Optimal Polygons***. The Mathematical Gazette, vol. 59, no. 409, oct-1975.
- [4] RIDLEY, J. N. ***A generalization of Reuleaux triangle***. Wits University.
- [5] BRYANT, J., SANGWIN, C. ***How Round is Your Circle?*** Princeton University Press, 2008, New Jersey.
- [6] KAVOHL, B., WEBER, C. ***Meissner's Mysterious Bodies***. Universit Mathematisches Institut, Köln, Germany, 2011.
- [7] MELLO, J. L. P. ***Polígonos de Reuleaux e a generalização de Pi***. Revista do Professor de Matemática (SBM), RJ, no. 81, 2º quadrimestre/2013