

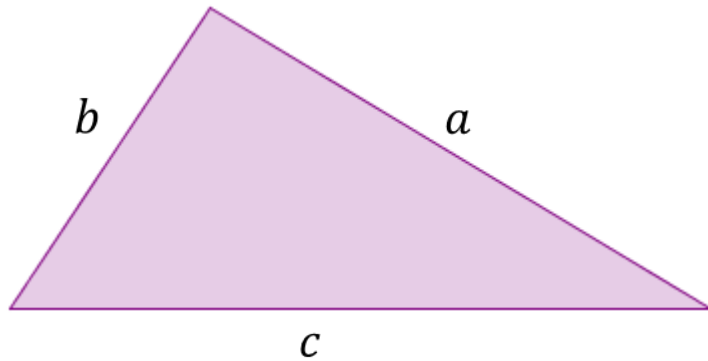
# A fórmula de Herão

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# A fórmula de Herão

A fórmula de Herão garante que, se  $a$ ,  $b$  e  $c$  são as medidas dos lados de um triângulo,

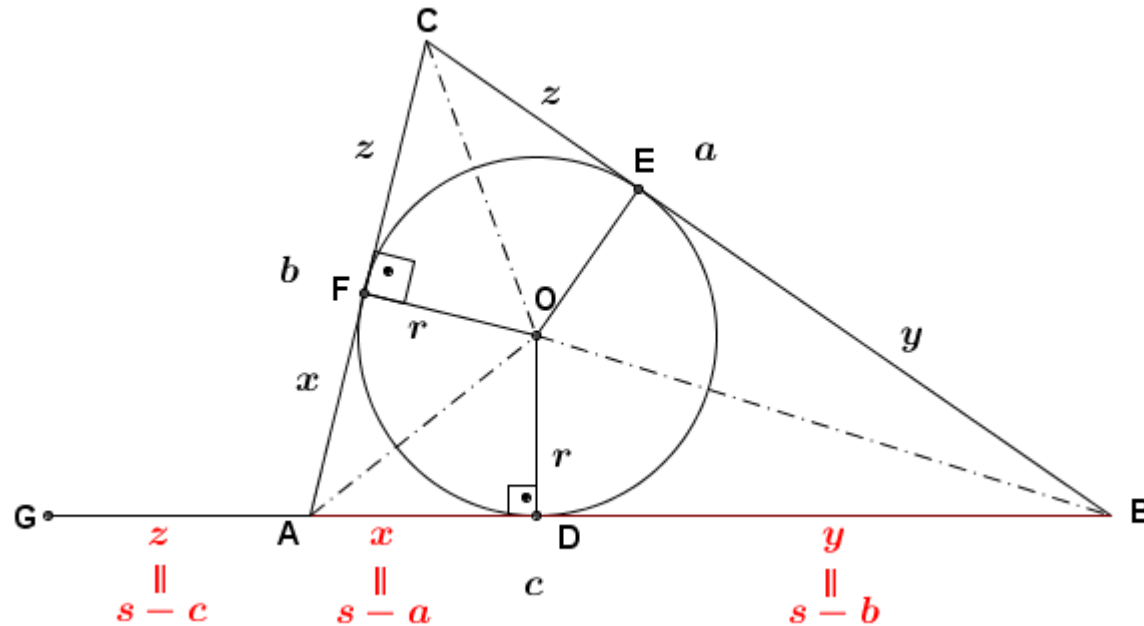


então sua área  $T$  pode ser expressa por

$$T = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

em que  $s$  é o semiperímetro do triângulo.

# A demonstração original



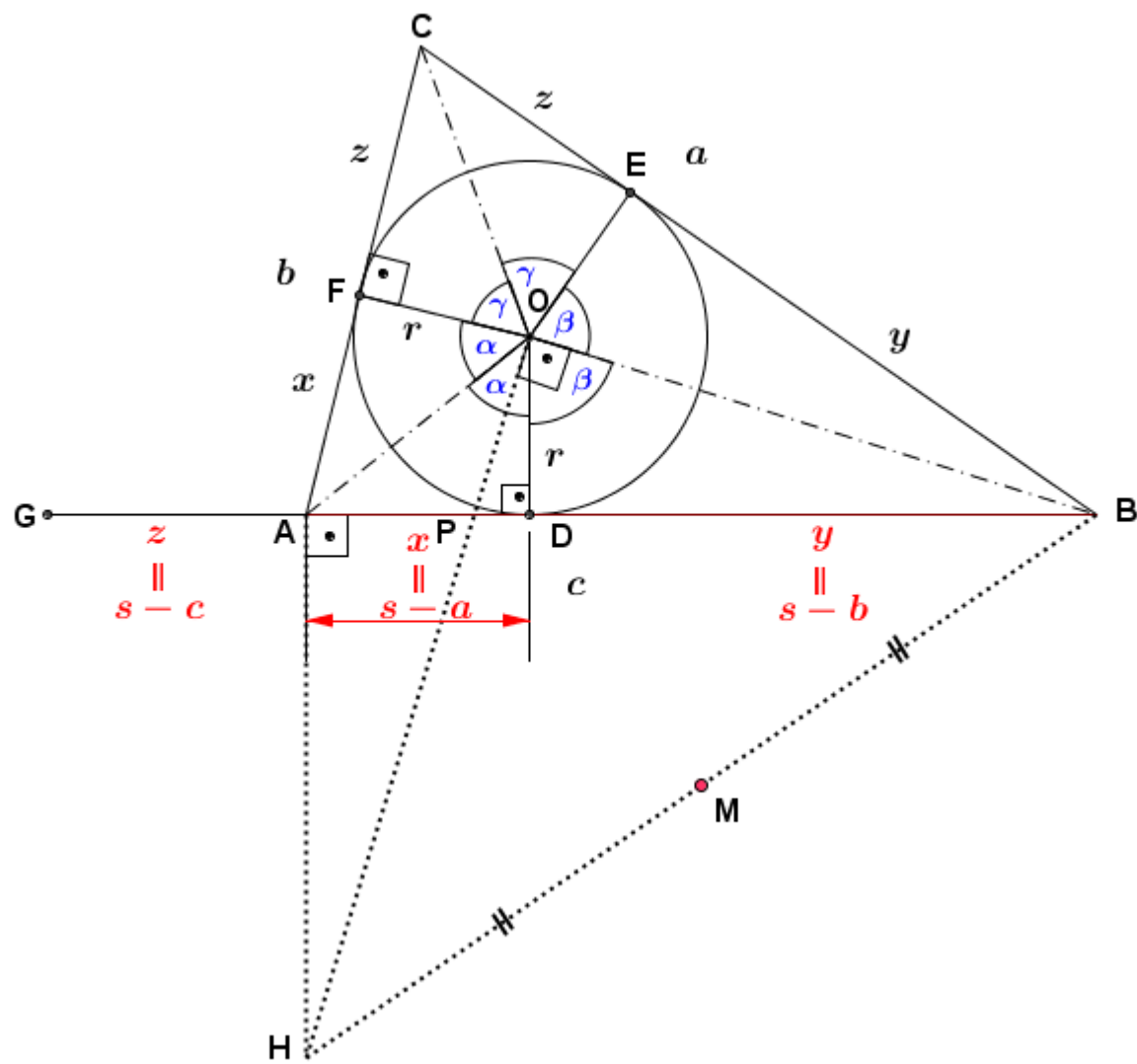
$$AB + BC + AC = 2x + 2y + 2z = 2s$$

$$x + y + z = s \Leftrightarrow x = s - (y + z) \Leftrightarrow x = s - a$$

$$x + y + z = s \Leftrightarrow y = s - (x + z) \Leftrightarrow y = s - b$$

$$x + y + z = s \Leftrightarrow z = s - (x + y) \Leftrightarrow z = s - c$$

$$\frac{s}{s-c} = \frac{s-a}{DP} \Leftrightarrow \frac{s}{s-c} = \frac{(s-a)(s-b)}{r^2} \Leftrightarrow r^2 s = (s-a)(s-b)(s-c)$$



$$r^2 s = (s - a)(s - b)(s - c)$$

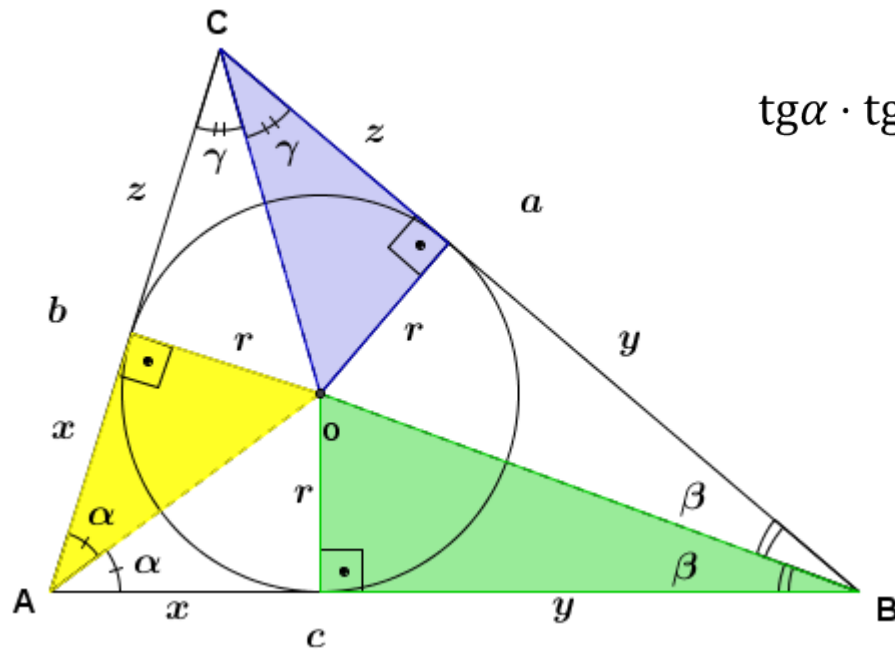
$$r^2 s^2 = s(s - a)(s - b)(s - c)$$

$$(rs)^2 = s(s - a)(s - b)(s - c)$$

$$(\textcolor{violet}{T})^2 = s(s - a)(s - b)(s - c)$$

$$T = \sqrt{s(s - a)(s - b)(s - c)}$$

*c. q. d.*



$$\operatorname{tg} \alpha \cdot \operatorname{tg} \beta + \operatorname{tg} \alpha \cdot \operatorname{tg} \gamma + \operatorname{tg} \beta \cdot \operatorname{tg} \gamma = 1$$

$$\frac{r}{x} \cdot \frac{r}{y} + \frac{r}{x} \cdot \frac{r}{z} + \frac{r}{y} \cdot \frac{r}{z} = 1$$

$$r^2 \cdot \left( \frac{1}{xy} + \frac{1}{xz} + \frac{1}{yz} \right) = 1$$

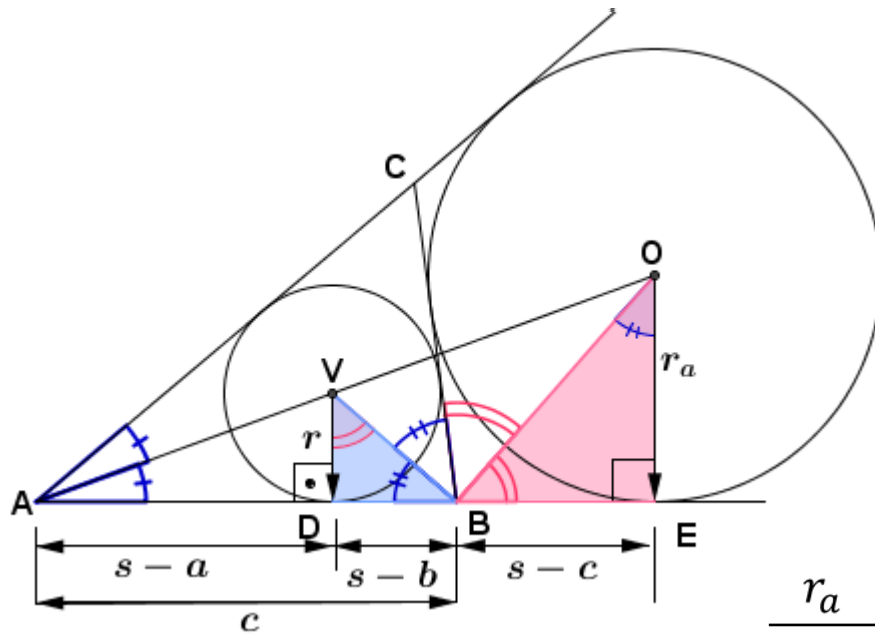
$$r^2 \cdot \left( \frac{z + y + x}{xyz} \right) = 1$$

$$r^2 \cdot (x + y + z) = xyz$$

$$r^2 s = (s - a)(s - b)(s - c)$$

$$(rs)^2 = s(s - a)(s - b)(s - c)$$

$$\therefore T = \sqrt{s(s - a)(s - b)(s - c)}$$



$$AB + BE = s \Leftrightarrow BE = s - c$$

$$\triangle AEO \approx \triangle ADV:$$

$$\frac{r_a}{r} = \frac{s}{s-a} \Leftrightarrow r \cdot s = r_a \cdot (s-a)$$


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$$\triangle OBE \approx \triangle BDV:$$

$$\frac{r_a}{s-b} = \frac{s-c}{r} \Leftrightarrow r \cdot r_a = (s-b)(s-c)$$


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$$r^2 \cdot s \cdot r_a = r_a(s-a)(s-b)(s-c) \Leftrightarrow r^2 \cdot s = (s-a)(s-b)(s-c)$$

$$r^2 s^2 = s(s-a)(s-b)(s-c) \Leftrightarrow T^2 = s(s-a)(s-b)(s-c)$$

$$\therefore T = \sqrt{s(s-a)(s-b)(s-c)}$$